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$SO(10)$ heterotic M-theory vacua *

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Abstract

This talk adapts the available formalism to study a class of heterotic M-theory vacua with $SO(10)$ grand unification group. Compactification to four dimensions with $\mathcal{N} = 1$ supersymmetry is achieved on a torus fibered Calabi-Yau 3-fold $\mathbf{Z} = \mathbf{X}/\tau_{\mathbf{X}}$ with first homotopy group $\pi_1(\mathbf{Z}) = \mathbb{Z}_2$. Here $\mathbf{X}$ is an elliptically fibered Calabi-Yau 3-fold which admits two global sections and $\tau_{\mathbf{X}}$ is a freely acting involution on $\mathbf{X}$. The vacua in this class have net number of three generations of chiral fermions in the observable sector and may contain M5-branes in the bulk space which wrap holomorphic curves in $\mathbf{Z}$. Vacua with nonvanishing and vanishing instanton charges in the observable sector are considered. The latter case corresponds to potentially viable matter Yukawa couplings. Since $\pi_1(\mathbf{Z}) = \mathbb{Z}_2$, the grand unification group can be broken with $\mathbb{Z}_2$ Wilson lines.

The motivation is to use the above formalism to extend realistic free-fermionic models to the nonperturbative regime. The correspondence between these models and $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification of the weakly coupled 10-dimensional heterotic string identifies associated Calabi-Yau 3-folds which possess the structure of the above $\mathbf{Z}$ and $\mathbf{X}$. A nonperturbative extension of the top quark Yukawa coupling is discussed.

1 Introduction

M-theory on the orbifold $\mathbf{S}^1/\mathbb{Z}_2$ is believed to describe the strong coupling limit of the $E_8 \times E_8$ heterotic string [1]. At low energy, this theory is described by 11-dimensional supergravity coupled to a chiral $\mathcal{N} = 1$, E_8 Yang-Mills supermultiplet on each of the two 10-dimensional orbifold fixed planes [2]. This low energy description is known as *Hořava-Witten theory*.

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Compactifications of Hořava-Witten theory leading to $\mathcal{N} = 1$ supersymmetry in four dimensions [3] are (to lowest order) based on the spacetime structure

$$\mathbf{M}^{11} = \mathbf{M}^4 \times \mathbf{S}^1/\mathbb{Z}_2 \times \boldsymbol{\mathcal{X}}, \tag{1.1}$$

where $\mathbf{M}^4$ is 4-dimensional Minkowski space and $\boldsymbol{\mathcal{X}}$ is a Calabi-Yau 3-fold. Generally, on a given fixed plane, some subgroup G of the E_8 symmetry survives on $\boldsymbol{\mathcal{X}}$. E_8 is broken to $G \times H$, where the grand unification group H is the commutant subgroup of G in E_8. The 'standard embedding', in which $G = SU(3)$ is identified with the spin connection of $\boldsymbol{\mathcal{X}}$, corresponds to $H = E_6$. Vacua with nonstandard embeddings may contain M5-branes [3, 4] in the bulk space. One refers to Hořava-Witten theory compactified to lower dimensions with arbitrary gauge vacua as *heterotic M-theory*.

Donagi, Ovrut, Pantev and Waldram [5] presented a class of heterotic M-theory vacua with $H = SU(5)$ grand unification group. The vacua in this class have net number of generations $N_{\rm gen} = 3$ of chiral fermions in the observable sector and may contain M5-branes in the bulk space at specific points in the orbifold direction. Compactification to four dimensions with $\mathcal{N} = 1$ supersymmetry is achieved on a *torus* fibered Calabi-Yau 3-fold

$$\mathbf{Z} = \mathbf{X}/\tau_{\mathbf{X}} \tag{1.2}$$

with first homotopy group

$$\pi_1(\mathbf{Z}) = \mathbb{Z}_2. \tag{1.3}$$

Here $\mathbf{X} \xrightarrow{\pi} \mathbf{B}$ is a torus fibered Calabi-Yau 3-fold which admits two global sections, a zero section σ and a second section ξ, and $\tau_{\mathbf{X}}$ is a freely acting involution on $\mathbf{X}$. A torus fibered manifold which admits a zero section is said to be *elliptically* fibered. Since $\pi_1(\mathbf{Z}) = \mathbb{Z}_2$, the $H = SU(5)$ grand unification group can be broken to the Standard Model gauge group with a $\mathbb{Z}_2$ Wilson line. The M5-branes are required to span $\mathbf{M}^4$ and wrap holomorphic curves in $\mathbf{Z}$.

This talk extends the above work by Donagi, Ovrut, Pantev and Waldram by considering the $H = SO(10)$ grand unification group. The $SO(10)$ embedding of the Standard Model spectrum is supported by experimental evidence [6] for neutrino masses. A class of string models which preserves this embedding are the *realistic free-fermionic models* [7, 8, 9, 10]. These $N_{\rm gen} = 3$, 4-dimensional string models are constructed using the free-fermionic formulation [11] of the weakly coupled heterotic string. The correspondence [12] between these models and $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification of the weakly coupled 10-dimensional heterotic string identifies associated Calabi-Yau 3-folds which possess the structure of the above $\mathbf{Z}$ and $\mathbf{X}$. This allows realistic free-fermionic models to be studied in the nonperturbative regime.

The new results presented in this talk have been published by Faraggi, Garavuso and Isidro [13] and Faraggi and Garavuso [14]. Section 2 reviews realistic free-fermionic models and their $\mathbb{Z}_2 \times \mathbb{Z}_2$ correspondence, explains the connection [13] with the Calabi-Yau 3-folds $\mathbf{Z}$ and $\mathbf{X}$, and offers a proposal to obtain a nonperturbative extension [14] of the top quark Yukawa coupling calculation

flipped $SU(5)$	$SO(10) \to SU(5) \times U(1)$
Pati-Salam	$SO(10) \to SO(6) \times SO(4)$
Standard-like	$SO(10) \to SU(3) \times SU(2) \times U(1)^2$
left-right symmetric	$SO(10) \to SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$

Table 2.1: $SO(10)$ breaking patterns in realistic free-fermionic models.

[15]. We will see that, although the precise geometrical realization of the full $N_{\text{gen}} = 3$ models is unknown, this coupling can be computed as a $\mathbf{27}^3$ E_6 or $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$ $SO(10)$ coupling with $N_{\text{gen}} = 24$. This leads us to present rules for constructing heterotic M-theory vacua allowing $H = E_6$ and $SO(10)$ grand unification groups with arbitrary N_{gen}. Section 3 summarizes rules [14] allowing grand unification groups $H = E_6$, $SO(10)$, and $SU(5)$, corresponding to $G = SU(n)$ with $n = 3$, 4, and 5, respectively. As discussed by Arnowitt and Dutta [16], requiring vanishing instanton charges in the observable sector yields potentially viable matter Yukawa couplings. $N_{\text{gen}} = 3$, $H = SO(10)$ vacua with nonvanishing [13] and vanishing [14] instanton charges in the observable sector are discussed in Section 4.

2 Realistic free fermionic models

2.1 General structure

A free-fermionic model is generated by a suitable choice of boundary condition basis vectors (which encode the spin structure of the worldsheet fermions) and generalized GSO projection coefficients. The boundary condition basis vectors associated with the realistic free-fermionic models are constructed in two stages. The first stage constructs the NAHE set [17] of five basis vectors denoted by $\{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. After generalized GSO projections over the NAHE set, the residual gauge group is

$$SO(10) \times SO(6)^3 \times E_8.$$

NAHE set models have $\mathcal{N} = 1$ spacetime supersymmetry and 48 chiral generations in the $\mathbf{16}$ representation of $SO(10)$ (16 from each of the sectors $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$). The sectors $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$ correspond to the three twisted sectors of the associated $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold. The second stage of the construction adds three (or four) basis vectors, typically denoted by $\{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \ldots\}$, which correspond to Wilson lines in the associated $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold formulation. These basis vectors break the $SO(10) \times SO(6)^3 \times E_8$ gauge group and reduce the number of chiral generations from 48 to 3 (one from each of the sectors $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$). The $SO(10)$ symmetry is broken to one of its subgroups. The flipped $SU(5)$ [7], Pati-Salam [8], Standard-like [9], and left-right symmetric [10] $SO(10)$ breaking patterns are shown in Table 2.1. In the former two cases, an additional $\mathbf{16}$ and $\overline{\mathbf{16}}$ representation of $SO(10)$ is obtained from the set $\{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \ldots\}$. Similarly,

the hidden E_8 is broken to one of its subgroups. The flavor $SO(6)$ symmetries are broken to flavor $U(1)$ symmetries. Three such symmetries arise from the subgroup of the observable E_8 which is orthogonal to $SO(10)$. Additional $U(1)$ symmetries arise from the pairing of real fermions. The final observable gauge group depends on the number of such pairings.

2.2 $\mathbb{Z}_2 \times \mathbb{Z}_2$ correspondence

The precise geometrical realization of the full $N_{\rm gen} = 3$ models is not yet known. However, the extended NAHE set $\{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \boldsymbol{\xi}_1\}$, or equivalently $\{\mathbf{1}, \mathbf{S}, \boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \mathbf{b}_1, \mathbf{b}_2\}$, has been shown to yield the same data as the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold of a toroidal Narain model [18] with nontrivial background fields [19]. This Narain model has $\mathcal{N} = 4$ spacetime supersymmetry and either

$$SO(12) \times E_8 \times E_8 \tag{2.1}$$

or

$$SO(12) \times SO(16) \times SO(16) \tag{2.2}$$

gauge group, depending on the choice of sign for the GSO projection coefficient $C\binom{\boldsymbol{\xi}_1}{\boldsymbol{\xi}_2}$. Let the former and latter Narain models be denoted by $\mathbf{N}_+$ and $\mathbf{N}_-$, respectively. The corresponding $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds are

$$\mathbf{Z}_+ \equiv \frac{\mathbf{N}_+}{\mathbb{Z}_2 \times \mathbb{Z}_2}, \quad \mathbf{Z}_- \equiv \frac{\mathbf{N}_-}{\mathbb{Z}_2 \times \mathbb{Z}_2}. \tag{2.3}$$

In the free-fermionic formulation, the $\mathbf{N}_+$ and $\mathbf{N}_-$ data is produced by the set $\{\mathbf{1}, \mathbf{S}, \boldsymbol{\xi}_1, \boldsymbol{\xi}_2\}$ with appropriate choices for the sign of $C\binom{\boldsymbol{\xi}_1}{\boldsymbol{\xi}_2}$. Adding the basis vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ corresponds to $\mathbb{Z}_2 \times \mathbb{Z}_2$ modding. These vectors reduce the spacetime supersymmetry to $\mathcal{N} = 1$, break

$$SO(12) \rightarrow SO(4)^3 \tag{2.4}$$

and either

$$E_8 \times E_8 \rightarrow E_6 \times U(1)^2 \times E_8 \tag{2.5}$$

or

$$SO(16) \times SO(16) \rightarrow SO(10) \times U(1)^3 \times SO(16). \tag{2.6}$$

The three sectors

$$\mathbf{b}_1 \oplus (\mathbf{b}_1 + \boldsymbol{\xi}_1), \quad \mathbf{b}_2 \oplus (\mathbf{b}_2 + \boldsymbol{\xi}_2), \quad \mathbf{b}_3 \oplus (\mathbf{b}_3 + \boldsymbol{\xi}_3)$$

correspond to the three twisted sectors of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, each sector producing eight generations in either the $\mathbf{27}$ representation of E_6 or $\mathbf{16}$ representation of $SO(10)$. The

$$\text{Neveu-Schwarz} \oplus \boldsymbol{\xi}_1$$

sector corresponds to the untwisted sector, producing an additional three $\mathbf{27}$ and $\overline{\mathbf{27}}$ or $\mathbf{16}$ and $\overline{\mathbf{16}}$ repesentations of E_6 or $SO(10)$, respectively.

As the NAHE set is common to all realistic free-fermionic models, $\mathbf{Z}_+$ and $\mathbf{Z}_-$ are at their core. To make a connection with the Calabi-Yau 3-folds $\mathbf{Z}$ and $\mathbf{X}$, we construct the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold at a generic point in the Narain moduli space. Start with the 10-dimensional space compactified on the torus $\mathbf{T}_1^2 \times \mathbf{T}_2^2 \times \mathbf{T}_3^2$ parameterized by three complex cooordinates z_1, z_2, and z_3, with the identification

$$z_i = z_i + 1, \quad z_i = z_i + \tau_i \qquad (i = 1, 2, 3) \tag{2.7}$$

where τ_i is the complex parameter of the torus $\mathbf{T}_i^2$. Under a $\mathbb{Z}_2$ twist $z_i \to -z_i$, the torus $\mathbf{T}_i^2$ has four fixed points at

$$z_i = \{0, 1/2, \tau_i/2, (1 + \tau_i)/2\}. \tag{2.8}$$

With the two $\mathbb{Z}_2$ twists

$$\alpha : (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \tag{2.9}$$

$$\beta : (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \tag{2.10}$$

there are three twisted sectors, α, β, and $\alpha\beta = \alpha \cdot \beta$, each with 16 fixed points and producing 16 generations in the $\mathbf{27}$ representation of E_6, for a total of 48. The untwisted sector produces an additional three $\mathbf{27}$ and $\overline{\mathbf{27}}$ representations of E_6. Denote this orbifold by $\mathbf{X}_+$. It corresponds to an elliptically fibered Calabi-Yau 3-fold $\mathbf{X}$ with $(h^{(1,1)}, h^{(2,1)}) = (51, 3)$ [20]. Now, add the shift

$$\gamma : (z_1, z_2, z_3) \to \left(z_1 + \frac{1}{2}, z_2 + \frac{1}{2}, z_3 + \frac{1}{2}\right). \tag{2.11}$$

The product of the γ shift with any of the three twisted sectors α, β, and $\alpha\beta$ does not produce any additional fixed points. Therefore, γ acts freely. Under the action of γ, the fixed points from the twisted sectors are identified in pairs. Hence, each twisted sector now has 8 fixed points and produces 8 generations in the $\mathbf{27}$ representation of E_6, for a total of 24. Consequently, this orbifold has $(h^{(1,1)}, h^{(2,1)}) = (27, 3)$, reproducing the data of $\mathbf{Z}_+$. Since γ acts freely, the associated Calabi-Yau 3-fold has $\pi_1 = \mathbb{Z}_2$ [21]. Thus, $\mathbf{Z}_+$ and $\mathbf{X}_+$ correspond to the Calabi-Yau 3-folds $\mathbf{Z}$ and $\mathbf{X}$, respectively.

2.3 Nonperturbative top quark Yukawa coupling

The Yukawa coupling of the top quark is obtained at the cubic level of the superpotential and is a coupling between states from the twisted-twisted-untwisted sectors. For example, in the Standard-like models [9], the relevant coupling is $t_1^c Q_1 \overline{h}_1$, where t_1^c and Q_1 are respectively the quark SU(2) singlet and doublet from the sector $\mathbf{b}_1$, and $\overline{h}_1$ is the untwisted Higgs. One can calculate this coupling in the full $N_{\text{gen}} = 3$ model, or at the level of either the $\mathbf{Z}_+$ or $\mathbf{Z}_-$ orbifolds as a $\mathbf{27}^3$ E_6 or $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10}$ $SO(10)$ coupling, respectively. The nonperturbative top quark Yukawa coupling at the grand unification scale is computed, at least in principle, as an integral over $\mathbf{Z}$ [22], with $\mathbf{Z}$ taken to be the Calabi-Yau 3-fold associated with $\mathbf{Z}_+$ or $\mathbf{Z}_-$.

3 Summary of rules

This section summarizes rules for constructing heterotic M-theory vacua with $H = E_6$, $SO(10)$, and $SU(5)$ grand unification groups and arbitrary $N_{\rm gen}$. For more details, refer to [14] or [23]. These rules are an adaptation of those presented in [5]. Compactification to four dimensions with $\mathcal{N} = 1$ supersymmetry is achieved on a torus-fibered Calabi-Yau 3-fold $\mathbf{Z} = \mathbf{X}/\tau_{\mathbf{X}}$ with first homotopy group $\pi_1(\mathbf{Z}) = \mathbb{Z}_2$. The $H = E_6$, $SO(10)$, and $SU(5)$ vacua correspond [24] to semistable holomorphic vector bundles $\mathbf{V}_{\mathbf{Z}}^{(1)}$ over $\mathbf{Z}$ having structure group $G_{\mathbb{C}} = SU(n)_{\mathbb{C}}$ with $n = 3$, 4, and 5, respectively. The vector bundles $\mathbf{V}_{\mathbf{Z}}^{(1)}$ and $\mathbf{V}_{\mathbf{Z}}^{(2)}$ are located on the observable and hidden orbifold fixed planes, $\mathbf{M}^{10}_{(1)}$ and $\mathbf{M}^{10}_{(2)}$, respectively. $\mathbf{V}_{\mathbf{Z}}^{(2)}$ is taken to be a trivial bundle so that E_8 remains unbroken in the hidden sector. The vacua with nonstandard embeddings may contain N M5-branes in the bulk space which wrap holomorphic curves $W_{\mathbf{Z}}^{(\ell)}$ $(\ell = 1, \ldots, N)$ in $\mathbf{Z}$. The class of the total M5-brane curve is denoted by $[W_{\mathbf{Z}}]$. $\beta_i^{(0)}$ and $\beta_i^{(N+1)}$ $(i = 1, \ldots, h^{(1,1)}(\mathbf{Z}))$ are the instanton charges on the observable and hidden orbifold fixed planes, respectively, and $\beta_i^{(\ell)}$ are the magnetic charges of the M5-branes. Requiring vanishing instanton charges in the observable sector yields potentially viable matter Yukawa couplings [16]. In a nutshell, the procedure is

1. Construct $\mathbf{Z} = \mathbf{X}/\tau_{\mathbf{X}}$.
2. Construct $\mathbf{V}_{\mathbf{Z}}^{(1)}$ over $\mathbf{Z}$ with structure group $G_{\mathbb{C}} = SU(n)_{\mathbb{C}}$.
3. $N_{\rm gen} = \frac{1}{2}\int_{\mathbf{Z}} c_3(V_{\mathbf{Z}}^{(1)})$
4. Require $[W_{\mathbf{Z}}] = c_2(\mathbf{TZ}) - c_2\big(\mathbf{V}_{\mathbf{Z}}^{(1)}\big) - c_2\big(\mathbf{V}_{\mathbf{Z}}^{(2)}\big)$ for anomaly cancellation.
5. Require $\beta_i^{(0)} = 0$.

The elliptically fibered Calabi-Yau 3-fold $\mathbf{X} \xrightarrow{\pi} \mathbf{B}$ admits global sections σ and ξ which satisfy $\xi + \xi = \sigma$. This condition facilitates the construction of the freely acting involution $\tau_{\mathbf{X}}$ as the composition $\tau_{\mathbf{X}} = \alpha \circ t_\xi$. Here, α is the lift to $\mathbf{X}$ of an involution $\tau_{\mathbf{B}}$ on the base $\mathbf{B}$, and $t_\xi(x) = x + \xi(b)$ $(x \in \pi^{-1}(b),\ b \in \mathbf{B})$ is a translation on the fiber $\pi^{-1}(b)$. The Calabi-Yau condition $c_1(\mathbf{TX}) = 0$ restricts the base [25] to be a del Pezzo $(\mathbf{dP}_r, r = 0, \ldots, 8)$, rational elliptic $(\mathbf{dP}_9)$, Hirzebruch $(\mathbb{F}_r, r \geq 0)$, blown-up Hirzebruch, or an Enriques surface.

A semistable holomorphic vector bundle $\mathbf{V}_{\mathbf{X}}^{(1)}$ over $\mathbf{X}$ with structure group $G_{\mathbb{C}} = SU(n)_{\mathbb{C}}$ can be constructed by the *spectral cover method* [26]. This method requires the specification of a divisor $\mathbf{C}$ of $\mathbf{X}$ (known as the *spectral cover*) and a line bundle $\boldsymbol{\mathcal{N}}$ over $\mathbf{C}$. The condition $c_1(\mathbf{V}_{\mathbf{X}}^{(1)}) = 0$ implies that the spectral data $(\mathbf{C}, \boldsymbol{\mathcal{N}})$ can be written in terms of an effective divisor class η in the base $\mathbf{B}$ and coefficients λ and κ_i $(i = 1, \ldots, 4\eta \cdot c_1(\mathbf{B}))$. Constraints are placed on η , λ , and the κ_i by the condition that

$$c_1(\boldsymbol{\mathcal{N}}) = n\left(\frac{1}{2} + \lambda\right)\sigma + \left(\frac{1}{2} - \lambda\right)\pi_{\mathbf{C}}^*\eta + \left(\frac{1}{2} + n\lambda\right)\pi_{\mathbf{C}}^* c_1(\mathbf{B}) + \textstyle\sum_i \kappa_i N_i \quad (3.1)$$

$\beta_i^{(0)} \neq 0$	$\exists\ \mathbf{B} = \mathbb{F}_2$ Class B vacua
	$\exists\ \mathbf{B} = \mathbb{F}_r$ (r even ≥ 4) Class B vacua*
	$\nexists\ \mathbf{B} = \mathbb{F}_0$ Class A or Class B vacua
	$\nexists\ \mathbf{B} = \mathbb{F}_r$ (r odd ≥ 1) Class A or Class B vacua
	$\nexists\ \mathbf{B} = \mathbf{dP}_3$ Class A or Class B vacua
$\beta_i^{(0)} = 0$	$\exists\ \mathbf{B} = \mathbf{dP}_7$ vacua*
	$\nexists\ \mathbf{B} = \mathbf{dP}_r$ ($r = 0, \ldots, 6, 8$) vacua
	$\nexists\ \mathbf{B} = \mathbb{F}_r$ ($r \geq 0$) vacua

Table 4.1: $H = SO(10)$ heterotic M-theory vacua with $N_{\text{gen}} = 3$. The '$*$' indicates that these vacua have been demonstrated to exist when certain constraints on $\tau_{\mathbf{B}}$ are satisfied.

be an integer class. To ensure that $c_1(\boldsymbol{\mathcal{N}})$ is an integer class, one can impose the sufficient (but not necessary) Class A or Class B constraints discussed in Section 4. More generally, $c_1(\boldsymbol{\mathcal{N}})$ will be an integer class if the constraints

$$q \equiv n\left(\frac{1}{2} + \lambda\right) \in \mathbb{Z} \tag{3.2}$$

$$\left(\frac{1}{2} - \lambda\right)\pi_{\mathbf{C}}^*\eta + \left(\frac{1}{2} + n\lambda\right)\pi_{\mathbf{C}}^* c_1(\mathbf{B}) \quad \text{is an integer class} \tag{3.3}$$

$$\kappa_i - \frac{1}{2}m \in \mathbb{Z}, \quad m \in \mathbb{Z} \tag{3.4}$$

are simultaneously satisfied. To ensure that H is the largest subgroup of E_8 preserved by $\mathbf{V}_{\mathbf{X}}^{(1)}$, impose the stability constraint [27] $\eta \geq nc_1(\mathbf{B})$ $(n \geq 2)$. Requiring $\tau_{\mathbf{X}}^*(\mathbf{V}_{\mathbf{X}}^{(1)}) = \mathbf{V}_{\mathbf{X}}^{(1)}$ ensures that $\mathbf{V}_{\mathbf{X}}^{(1)}$ descends to a bundle $\mathbf{V}_{\mathbf{Z}}^{(1)}$ over $\mathbf{Z}$.

4 $N_{\text{gen}} = 3$, $H = SO(10)$ vacua

$N_{\text{gen}} = 3$, $H = SO(10)$ heterotic M-theory vacua with nonvanishing and vanishing instanton charges in the observable sector are searched for in [13] and [14], respectively. In the former work, the search is restricted to vector bundles satisfying the Class A or Class B constraints:

$$\text{Class A:} \quad \lambda \in \mathbb{Z}, \quad \eta = c_1(\mathbf{B}) \bmod 2, \quad \kappa_i - \frac{1}{2}m \in \mathbb{Z} \tag{4.1}$$

$$\text{Class B:} \quad \lambda - \frac{1}{2} \in \mathbb{Z}, \quad c_1(\mathbf{B}) \text{ is an even class}, \quad \kappa_i - \frac{1}{2}m \in \mathbb{Z} \tag{4.2}$$

where $m \in \mathbb{Z}$. In the latter work, the more general constraints (3.2), (3.3), and (3.4) with $n = 4$ are imposed. Table 4.1 summarizes the results.

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